Physics 238C **Spring 2011**

Condensed Matter Physics **due 2:00 pm Thursday May 12**

PROBLEM SET 6

Reading: AM Chapter 34 or Marder (2nd Ed) Chapter 27 or Kittel (8th Ed) Chapter 10

1. **AC Josephson Effect** When a static DC voltage V is applied across a Josephson junction, an AC current results. To see how this comes about, notice that an electron pair experiences a potential energy difference qV on passing across the junction, where $q = -2e$. We can say that a pair on one side is at potential $-eV$ and a pair on the other side is at $+eV$. Thus the equations of motion become

$$i\hbar\frac{\partial\psi_1}{\partial t} = \hbar T\psi_2 - eV\psi_1 \qquad i\hbar\frac{\partial\psi_2}{\partial t} = \hbar T\psi_1 + eV\psi_2 \tag{1}$$

where ψ_1 is the superconducting order parameter on side 1:

$$\psi_1 = \sqrt{n_1}e^{i\theta_1} \tag{2}$$

n_1 is the density of superconducting pairs on side 1. Similarly

$$\psi_2 = \sqrt{n_2}e^{i\theta_2} \tag{3}$$

Assume that the superconductors are identical. Find the current density J as a function of time and of the phase difference $\delta(0)$. $\delta(0) = \theta_2 - \theta_1$ is the phase difference at $V = 0$. What is the angular frequency ω at which the current oscillates when a voltage V is applied?

2. **Diffraction effect of a Josephson junction**: Consider a junction of square cross section with a magnetic field B applied in the plane of the junction. Each side of the square has width a. Let the thickness of the junction be d. Assume for convenience that the phase difference of the two superconductors is $\pi/2$ when $B = 0$. Show that the DC current in the presence of the magnetic field is

$$I = I_c\frac{\sin\left(adBe/\hbar c\right)}{adBe/\hbar c} \tag{4}$$